Design of an Attitude Control System with Magnetometer Sensors

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Introduction

THIS note presents the design of an attitude control system using magnetometers for attitude intelligence. The system is unique in that it is designed to capture a vehicle from any initial unknown attitude and motion in order to establish a specific known attitude with respect to the earth. Magnetometers have been used for some time in artificial satellites for measurement of the earth's field. However, only recently has this instrument been considered for attitude sensing in a closed loop control system. Consider the case of a vehicle tumbling in space either due to loss of intelligence or impulse control, or one requiring control only in the terminal phase of the mission. The failure of almost any element in an attitude control system so as to temporarily lose earth reference normally would result in mission failure since horizon scanners, star trackers, and sun sensors have a limited field of view. A system able to establish a known attitude reference for reinstating inertial reference would be highly desirable in order to avoid mission failure. An application like this might well be used as an emergency control system for Mercury or Gemini.

A schematic diagram of the attitude control system is shown in Fig. 1. The system presented is designed to perform this reacquisition function by aligning the vehicle longitudinal axis parallel to the local magnetic field vector, which is defined well with respect to the earth, at any given location. For some applications, this capability alone is sufficient and no roll attitude information is required. The general case, however, requires specific orientation of the vehicle about its roll axis and a simple tristable infrared horizon sensor would be added to the mechanization shown in Fig. 1.

The magnetometer is characteristically a device whose output is proportional to the cosine of the angle between the field vector and the instrument-sensitive axis. It is desired to use the instruments for null sensing and the pitch and yaw magnetometers therefore should be mounted perpendicular to each other such that their output is proportional to the sine of the angle between the vehicle roll axis and the magnetic field vector (H) in the plane of the instruments.

In the application presented here, consisting of orienting a symmetric vehicle in the direction of the local magnetic field vector without a roll attitude requirement, the designation of pitch and yaw axis becomes arbitrary. However, for the purposes of design, analysis, and test, it is convenient to speak of the pitch and yaw axis. The vehicle is a rotating coordinate system and the magnetometers perform their functions of resolving the vector quantity $H \sin\alpha$ into the pitch and yaw channels regardless of roll attitude. Although roll position information is not required for attitude reference, it will be shown that roll control is necessary in this system to assure stable dynamic performance.

The use of magnetometers for attitude sensors offers particular advantages in view of the design criteria. First, the magnetometer coupled with the earth's field establishes a reference direction regardless of vehicle orientation and rate without the need for erection or knowledge of initial conditions. It is this unique capability that makes the system

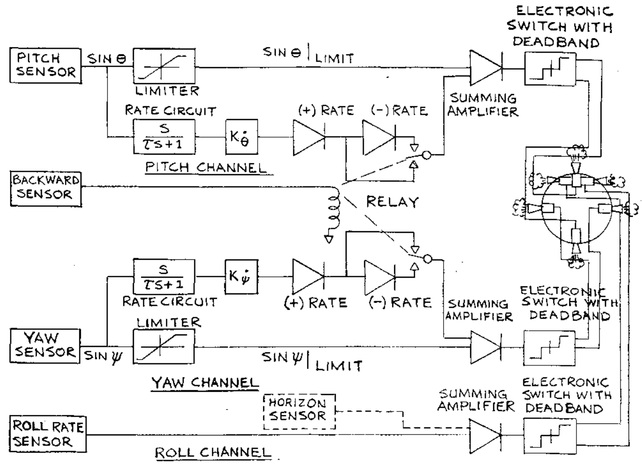

Fig. 1 Schematic of three axis on-off attitude control system with rate reversal and attitude limit

possible. Second, the magnetometer has no moving parts. The associated electronics are minimum, commensurate with the weight and packaging requirements.

To provide system damping for stable operation requires some type of attitude lead information. This could be provided either by rate gyros or lead circuits. Although certain dynamic difficulties develop with the use of lead circuits in this application, this method was selected to provide damping information on the basis of its inherent reliability.

Following analog simulation of the control system and vehicle dynamics, it was found that even small roll rates ($>5°$/sec) contaminated the differentiated pitch and yaw rate signals destabilizing the operation. This effect was not due to gyroscopic coupling, but a phenomenon directly due to the use of rate circuits that detect significant effects of even relatively small roll rates that degrade performance. Therefore, it was found necessary to add a roll channel to the design, which uses a rate gyro for control measurement.

Control System and Operation

The system evolved to satisfy the design uses a three axis contactor or on-off servo. The system consists of identical pitch and yaw channels that use attitude and rate circuit feedback signals and a roll channel using a rate gyro for control measurement.

The instrumentation for this attitude control system measures the "components" of pitch and yaw angles from the reference direction. In both the pitch and yaw channels, the attitude signal is differentiated and limited independently. The limited position signal and the rate signal are then summed. When the summed autopilot signal exceeds the present threshold value (deadband, θ_D) the control force is actuated. The sign of both the pitch and yaw rate feedback is reversed when the vehicle's X axis is pointing rearward from the reference direction. The third magnetometer is used to sense this backward condition and is mounted along the vehicle axis.

The roll channel is even simpler than those for pitch and yaw. The output of the roll rate gyro is connected directly through suitable amplification to an electronic deadband switch that actuates the roll nozzles. The nozzle configuration in Fig. 1 shows that the roll forces are unbalanced. A control force to correct roll rate gives a torque disturbance into the yaw channel. This effect is not harmful since the greatest use of the roll actuator is within the first few seconds of system operation and rarely is used thereafter provided the ratio of torque to moment of inertia about the roll axis of the vehicle is large compared to the torque to moment of inertia about the pitch or yaw axis. The effect is further minimized by moving the roll valves forward toward the center of mass. The advantage of the unbalanced roll actu-

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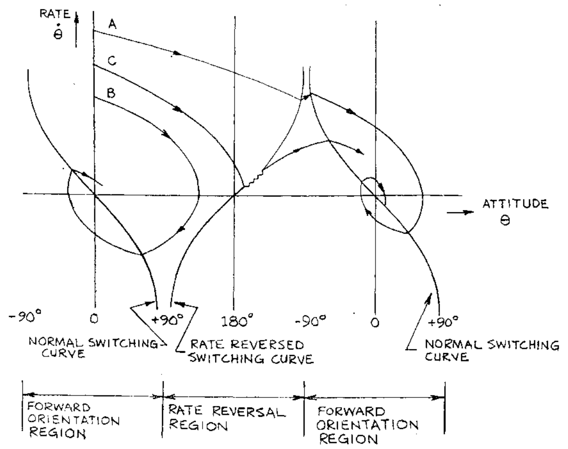

Fig. 2 Phase plane of contactor servo with rate reversal

ators is to allow a simpler system mechanization with completely isolated roll instrumentation and electronics.

The switching function for an ideal lead circuit operation on a device that gives the sine of the attitude error is

$$\theta_D = \sin\theta + K_{\dot\theta}(d/dt)(\sin\theta)$$
$$= \sin\theta + K_{\dot\theta}\cos\theta\dot\theta$$

or

$$\dot\theta = -(1/K_{\dot\theta})[\tan\theta - (\theta_D/\cos\theta)]$$

The attitude sensing instruments that give a sinusoidal function of the attitude error have a false null at 180°. The bang-bang system cannot stabilize at this point. However, unless prevented, the system needlessly would accelerate the vehicle away from the backward orientation and in the process build up the body rate that must be decelerated with considerable consumption of the stored fuel.

An arrangement that produces a more efficient control operation provides an isolated rate signal and a sign changing amplifier along with a relay for switching in either sign of the rate in the final summing amplifier. The switching signal is to come from the backward orientation sensing magnetometer. When the rate feedback is reversed, the sign of the equation changes. Figure 2 shows a phase plane neglecting the small deadband for the system with the rate gain reversed for attitudes in the backward quadrants.

There are three kinds of trajectories to consider with the rate reversal system. Referring to Fig. 2, case A has a large initial rate and the body tumbles end-over-end at least once. Note that the accelerate forward arc of the trajectory, after switching at the rate-reversed switching curve, is fairly short compared to the entire capture phase of operation. Case B does not have a large initial rate and does not cross the rate-reversed switching curve. Note that the attitude excursion can approach 180° and still be captured without end-over-end tumbling. Case C has an intermediate initial rate and its trajectory of motion intersects the rate-reversed switching curve at a low value of rate. The motion that follows is a reticent or staircasing operation, as shown in the Fig. 2. The staircase increases the rate until the parabola of the trajectory tears away from the switching curve. When the slope of the trajectory is tangent to or less than that of the switching curve, no re-intersection is possible. Following this staircase divergence away from the 180° point, the capture and control motion would continue as in case B.

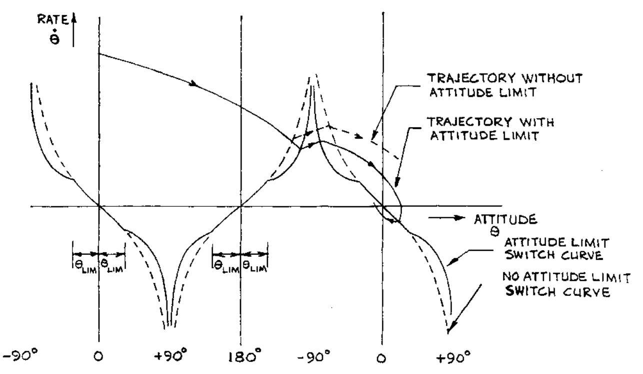

Fig. 3 Phase plane of contactor servo with rate reversal and attitude limit

A further improvement for the capture of high initial rates can be achieved by limiting the attitude error. The effect of this is to distort the switching curve so that the converging trajectories can continue without switching for longer periods than the previous schemes allow. The switching curve becomes

$$\dot\theta = -\frac{1}{K_{\dot\theta}}\left[\frac{\sin\theta\,|_{\rm lim} - \theta_D}{\cos\theta}\right]$$

where

$$\sin\theta\,|_{\rm lim} = \sin\theta \text{ for } |\sin\theta| < \theta_{\rm lim}$$
$$\pm\theta_{\rm lim} \text{ for } |\sin\theta| > \theta_{\rm lim}$$

A comparison of the switching curves with and without attitude limiting curves is shown in Fig. 3. Note the improved convergence of the trajectory with attitude limit. In general, the gas impulse for acquisition is reduced with decreases in the attitude limit but the time to converge increases. The limiting case for decreasing the attitude limit is the removal of position control for which the system would stop any vehicle motion, but never orient the vehicle in the desired direction.

The influence of minor variations in system gains and hardware nonlinearities did not influence greatly the acquisition or capture phase of motion. The anomalies of operation of the actuation, such as hysteresis and time lags, are of particular significance only in determining the steady-state limit cycle.

Roll Effects

As previously mentioned, the analog investigation of an attitude control system using differentiating networks for rate signals showed that unsatisfactory control performance resulted from the presence of roll rates higher than a few degrees per second. A high percentage of initial vehicle orientations could not be brought under control if significant roll rates existed (greater than 5°/sec) and those that were controllable required an excessive amount of control impulse. This phenomenon would not exist if rate gyros were used in place of the lead circuits.

For small angles (up to about 45°) the following approximations of the attitude feedback terms are valid:

$$\bar\theta = \theta\cos\eta + \psi\sin\eta$$
$$\bar\psi = \psi\cos\eta - \theta\sin\eta$$

and the differentiated terms are

$$\dot{\bar\theta} = \dot\theta\cos\eta + \dot\psi\sin\eta + \overbrace{[\psi\cos\eta - \theta\sin\eta]}^{\bar\psi}P$$
$$\dot{\bar\psi} = \dot\psi\cos\eta - \dot\theta\sin\eta - \underbrace{[\theta\cos\eta + \psi\sin\eta]}_{\bar\theta}P$$

where

$\bar\theta$ = inertial pitch angle
ψ = inertial yaw angle
η = roll angle
p = roll rate
$\bar\theta, \bar\psi$ = sensed pitch and yaw in the rolling body

The bracketed terms in the differentiated expressions are proportional to roll rate and represent the cross coupling that proved disastrous to the control performance. Any compensation for this coupling effect requires information on the size of roll rate. It was decided that the best compensation for the deleterious effects would be the removal of the effect, e.g., a roll control channel. The roll control channel reduces roll rate to an acceptably low level (<2° sec).

With the decision to control roll rate in the vehicle, the analysis and understanding of planer on-off control systems can be applied to the problem.[1-3] With roll rate control, the vehicle spin rate is controlled to be within a threshold value, p, within a few seconds of system activation and the vehicle is brought under control or acquired for any arbitrary orientation. This overall stability did not exist in the system without roll control.

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Turbulence Effects in Chemical Reaction Kinetics Measurements

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I. Introduction

RECENT interest in the effect of turbulent flow on reaction kinetics has developed from consideration of such problems as recombination of atoms in hypersonic wakes,[1] efficient supersonic combustion,[2] etc. The authors' interest in the problem stems from their use of a flow reactor for the measurement of chemical reaction rates. This reactor is different from the classical flow reactor in that the gas flow is turbulent.[3] In the course of examining possible sources of errors that may arise in chemical kinetic measurements due to turbulent fluctuations, some simple phenomenological developments were evolved which could be of interest to other investigators. The developments are quite different from the detailed turbulent flow approaches of other investigators;[4, 5] however, they bring to light more clearly some of the important physical aspects of the problem.

II. Aspects of the Problem

Turbulence may affect the reaction rate in two ways. It may alter the fundamental chain mechanism either by selectively enhancing various elementary reactions of the chain or by altering the diffusion of free radicals. The second way, which is really a simplification of the first, is enhancement of the overall rate, the instantaneous value of which may be expressed in the form

$$\nu = dc/dt = -kc^n = -c^n B T^m e^{-E/RT} \qquad (1)$$

Thus the problem is first to ascertain whether or not the basic mechanism is affected by the turbulence. If this is not the case, and if steady-state kinetics do apply to the turbulent field, one must determine whether the rate at the means of the fluctuating quantities (T and c) differs significantly from the mean rate.

A third element of the problem is whether the turbulence enhances the eddy characteristics to a point where, for a one-dimensional system, longitudinal heat and mass transfers are such that they contribute to the temperature or concentration at a point and not just to the chemical reaction alone. This point is a simple extrapolation of the laminar flame propagation problem, where the temperature in the flame front is determined by the extent of the chemical reaction and by the amount of heat conducted back from the high temperature edge of the flame.[6] Only the laminar conductivity must be replaced by a turbulent eddy conductivity. If the effect of heat transfer is small, the adiabatic temperature profile may be related to the measured temperature profile by

$$\left(\frac{dT}{dx}\right)_{\text{ad}} = \left(\frac{dT}{dx}\right)_{\text{meas}} \left\{1 + \left[\frac{d^2T/dx^2}{(\dot{m}C_p/\epsilon A)(dT/dx)}\right]\right\} = \left(\frac{dT}{dx}\right)_{\text{meas}} (1 + \beta)$$

where β is the ratio of conductive to convective terms. For the adiabatic flow reactor mentioned previously, conditions are such that β is less than 1%, and conductivity effects may be neglected.[7]

III. Steady-State Considerations

In the description of turbulence given by Hinze,[8] flow in the smallest eddies is no longer turbulent but viscous, and molecular effects are dominant. Batchelor[9] states that the energy of turbulent motion dies away effectively to zero long before length scales comparable with the molecular mean free path are reached. It seems reasonable to assume from Refs. 8 and 9 that the only means by which the chemical kinetics within the smallest eddies of the turbulent field can be affected are turbulent pressure fluctuations in the fluid and molecular diffusion of species into or out of the tiny eddy. If the mean distance travelled by a tiny laminar pocket during its lifetime is sufficiently small so that it does not travel into regimes whose concentration and temperature are drastically different, then it will not encounter steep gradients, and the rate of change of conditions in the eddy due to diffusion effects will be slow.

For flow in a circular pipe, such an eddy, "lifepath" l is likely to be less than the pipe diameter D, and, for Reynolds numbers used in the flow reactor ($Re \sim 10{,}000$), l is approximately 0.2 D.[10] From these criteria, it is found that in the flow reactor an eddy encounters a 2% change in concentration and a 2°C difference in temperature. It seems safe to say that these gradients are sufficiently shallow so that diffusion will not cause any rapid fluctuations of temperature or concentration within an eddy. As regards pressure fluctuations and temperature fluctuations due to compressibility, Wight[11] has shown that small amplitude fluctuations do not affect significantly the chemical kinetics. From information given by Lewis and von Elbe[10] and Laurence,[12] a characteristic time of turbulent oscillation may be estimated. For conditions in the flow reactor, a characteristic turbulent time of 20 to 30 msec is obtained.

A mechanism for the thermal decomposition of hydrazine was proposed,[7] and the resulting set of differential equations was integrated numerically. The results that are presented in Fig. 1 show that the time necessary to reach a chemical steady state is generally less than 0.1 msec when the temperature is in the 1000°K range. Under these conditions, the chemical transient time is only 0.5% of the characteristic time of turbulent fluctuation just given. Actually, the time

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